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SPACETIME CURVATURE FROM RESIDUAL MICROSCOPIC INTERACTIONS: AN EMERGENT-GRAVITY APPROACH BASED ON SPIN AND BINDING FORCES

Wanjohi N.P.

Kirinyaga University, KENYA

Correspondences: pnjori@kyu.ac.ke

Abstract

Emergent gravity proposes that gravitation is not a fundamental interaction but rather a macroscopic manifestation of underlying microscopic degrees of freedom. Existing approaches, including thermodynamic gravity, entropic gravity, and holographic gravity, provide important insights into the relationship between information, statistical mechanics, and space-time geometry. However, a direct mathematical framework connecting microscopic particle interactions, spin effects, binding forces, and space-time curvature remains incomplete.

This paper presents a theoretical framework in which residual microscopic interactions generate an effective scalar field whose stress-energy contributes to space-time curvature. A residual interaction density is constructed from electromagnetic, strong, weak, spin, and vacuum contributions. Using variational methods based on the Einstein–Hilbert action, modified Einstein field equations are derived and proof is provided that non-vanishing residual interactions necessarily induce curvature. The framework establishes a mathematical bridge between microscopic interaction physics and macroscopic gravitational geometry, providing a potential foundation for emergent gravity rooted in particle-level dynamics.

Keywords: Emergent gravity, spacetime curvature, Einstein field equations, scalar field theory, residual interactions, particle spin, binding energy, quantum vacuum, variational principles, mathematical physics.

1. Introduction

1.1 Background

The nature of gravity remains one of the most profound questions in theoretical physics. While the other fundamental interactions are described within the framework of quantum field theory, gravity is successfully modeled by Einstein's General Theory of Relativity, where gravitational phenomena arise from the curvature of spacetime. This is well described by Einstein tensor equation:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu}, \tag{1}$$

Where,

- $G_{\mu\nu}$ (Einstein tensor) is a symmetric tensor that combines information about the curvature of a spacetime.
- $R_{\mu\nu}$ is the Ricci curvature tensor representing how much space time is curved?
- R is the scalar curvature the trace of Ricci tensor?
- $g_{\mu\nu}$ is the metric tensor, describing the geometry of spacetime.

The Einstein tensor ($G_{\mu\nu}$) ombines the Ricci tensor and Ricci scalar to produce a divergence-free tensor that encodes the curvature of spacetime in a form suitable for Einstein's field equations:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2}$$

where

- G is the Newtonian's gravitational constant?
- $T_{\mu\nu}$ is the stress-energy tensor, describing the distribution and flow of energy and matter.

Despite its profound success in describing gravitational phenomena at macroscopic scales, General Relativity does not account for the microscopic origins of gravity. This limitation has led to the development of emergent-gravity theories, which propose that gravity arises as a collective effect stemming from fundamental microscopic processes. Additionally, quantum mechanics indicates that all known physical phenomena originate from interactions among elementary particles, suggesting that gravity may also be an emergent phenomenon rooted in microscopic interactions. In this context, this work explores the hypothesis that residual microscopic interactions—such as those involving particle spin, electromagnetic binding, strong nuclear confinement, weak interactions, and quantum vacuum fluctuations—collectively contribute to an effective gravitational effect manifested through spacetime curvature.

1.2 Emergent Gravity

Emergent gravity theories propose that gravity is not fundamental but arises from collective microscopic behavior. Examples include:

i. Thermodynamic gravity (Jacobson, 1995)
ii. Entropic gravity (Verlinde, 2011)
iii. Holographic gravity and
iv. Quantum information approaches.

These theories suggest that spacetime geometry may emerge from microscopic structures in a manner analogous to thermodynamic laws emerging from molecular dynamics.

However, most existing models focus on entropy and quantum information. The possible role of microscopic interaction forces, particle spin, binding energies, and vacuum effects remains less developed.

1.3. Objective

The objective of this work is to formulate a mathematical framework in which residual microscopic interactions produce spacetime curvature through an emergent scalar field.

The study specifically seeks to establish the chain

2. Mathematical Preliminaries

2.1 Spacetime Geometry

Let $(M, g_{\mu\nu})$ be a four-dimensional Lorentzian spacetime manifold, where $g_{\mu\nu}$ is the metric tensor.

The metric determines:

i. distances,
ii. angles,
iii. causal structure and
iv. spacetime geometry.

2.2. Residual Interaction Field

Let $F_R(x)$ denote the residual microscopic interaction density arising from the collective contributions of

$F_{em}, F_s, F_w, F_{spin}, F_{vac}$, namely,

$$F_R = F_{em} + F_s + F_w + F_{spin} + F_{vac}. \tag{3}$$

This field represents the net microscopic interaction proposed to generate emergent gravity.

2.3. Emergent Scalar Field

It is postulated that the residual interactions generate a macroscopic scalar field Φ.

Mathematically,

$$\Phi = \Phi(F_R). \tag{4}$$

Thus,

$$F_R \longrightarrow \Phi. \tag{5}$$

2.3. Einstein Tensor

The Einstein tensor is

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \tag{6}$$

Where

- $R_{\mu\nu}$ is the Ricci curvature tensor and
- $R = g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar.

The Einstein tensor completely characterizes spacetime curvature.

2.4. Stress-Energy Tensor

The stress-energy tensor measures:

i. energy density,

ii. momentum density,

iii. pressure and

iv. stresses.

For ordinary matter (m), stress-energy tensor is given as:

$$T_{\mu\nu}^{m}(m). \tag{7}$$

For the scalar field (Φ) , it is given as:

$$T_{\mu\nu}^{(\Phi)}. \tag{8}$$

2.5. Variational Formulation

Total Action

The total action is

$$S = S_{grav} + S_{\Phi} + S_{m}, \tag{9}$$

3.0. Main Theorem

3.1. Theorem 1: Residual Microscopic Interactions Generate Spacetime Curvature

3.2. Statement:

Let $F_R(x)$denote a residual microscopic interaction field generated by the collective effects of particle spin, electromagnetic binding, strong nuclear confinement, weak interactions, and quantum vacuum fluctuations.
Suppose there exists a differentiable scalar field:

$$\Phi = \Phi(F_R), \tag{10}$$

induced by the residual interaction field. Then the energy and momentum carried by (Φ) contribute to the Einstein field equations and consequently generate spacetime curvature.

3.3. Proof

3.3.1. Construct the Total Action

Considering the Einstein-Hilbert action coupled to the scalar field,

The total action is

$$S = S_{grav} + S_{\Phi} + S_{m},$$

Where

$$S_{grav} = \frac{1}{16\pi G}\int_{M} R\sqrt{-g}\, d^{4}x, \tag{11}$$

$$S_{m} = \int_{M} \mathcal{L}_{m}\sqrt{-g}\, d^{4}x, \tag{12}$$

And

$$S_\Phi = \int_M \left[-\frac{1}{2}\nabla_\mu \Phi \nabla^\mu \Phi - V(\Phi)\right]\sqrt{-g}\, d^4x. \tag{13}$$

Meaning of the Terms

$\boldsymbol{G}$	Newton's gravitational constant.
$\boldsymbol{R}$	Ricci scalar curvature.
$\sqrt{-\boldsymbol{g}}$	Invariant spacetime volume element.
$\boldsymbol{V(\Phi)}$	Potential energy of the scalar field.
$\boldsymbol{\nabla_\mu}$	Covariant derivative.

3.3.2. Apply Hamilton's Principle

Hamilton's principle states that physical solutions satisfy:

$$\delta S = 0. \tag{14}$$

Varying the action with respect to the metric tensor, $g_{\mu\nu}$.

Thus,

$$\delta S = \delta S_{grav} + \delta S_\Phi + \delta S_m. \tag{15}$$

3.3.3. Variation of the Einstein-Hilbert Action

A fundamental theorem of General Relativity gives:

$$\delta S_{grav} = \frac{1}{16\pi G}\int_M G_{\mu\nu}\delta g^{\mu\nu}\sqrt{-g}\, d^4x. \tag{16}$$

This result follows from the variation of the Ricci scalar.

Hence the Einstein tensor naturally emerges from the variational principle.

3.3.4. Variation of the Matter Action

The stress-energy tensor is defined by:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}. \tag{17}$$

Therefore,

$$\delta S_m = -\frac{1}{2}\int_M T_{\mu\nu}^{(m)}\delta g^{\mu\nu}\sqrt{-g}d^4x. \tag{18}$$

3.3.5. Variation of the Scalar Field Action

Similarly,

$$\delta S_{\Phi} = -\frac{1}{2}\int_{M} T_{\mu\nu}^{(\Phi)}\ \delta g^{\mu\nu}\sqrt{-g}\, d^4x. \tag{19}$$

The scalar-field stress-energy tensor is:

$$T_{\mu\nu}^{(\Phi)} = \nabla_{\mu}\Phi\nabla_{\nu}\Phi - g_{\mu\nu}[\frac{1}{2}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi + V(\Phi)]. \tag{20}$$

Interpretation of the Scalar Stress-Energy Tensor

The first term;

$$\nabla_{\mu}\Phi\nabla_{\nu}\Phi, \tag{21}$$

represents the kinetic energy contribution.

The second term;

$$\frac{1}{2}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi + V(\Phi). \tag{22}$$

represents:

i. potential energy,

ii. pressure and

iii. isotropic stress.

Thus the scalar field carries energy and momentum.

3.3.6. Combine All Variations

Substituting into

$$\delta S = 0,$$

gives:

$$\frac{1}{16\pi G}\int_{M} G_{\mu\nu}\delta g^{\mu\nu}\sqrt{-g}\, d^4x - \frac{1}{2}\int_{M} T_{\mu\nu}^{(m)}\delta g^{\mu\nu}\sqrt{-g} d^4x - \frac{1}{2}\int_{M} T_{\mu\nu}^{(\Phi)}\ \delta g^{\mu\nu}\sqrt{-g}\, d^4x = 0. \tag{23}$$

This reduces to

$$\left(G_{\mu\nu}\delta g^{\mu\nu}\sqrt{-g}\right)\frac{1}{16\pi G}\int_M d^4x\ -\frac{1}{2}T_{\mu\nu}^{(m)}\delta g^{\mu\nu}\sqrt{-g}-T_{\mu\nu}^{(\Phi)}\delta g^{\mu\nu}\sqrt{-g}\int_M d^4x=0. \tag{24}$$

$$\therefore\left[\frac{G_{\mu\nu}}{16\pi G}-\frac{1}{2}T_{\mu\nu}^{(m)}-\frac{1}{2}T_{\mu\nu}^{(\Phi)}\right]\int_M d^4x=0. \tag{25}$$

$$\Rightarrow\left[\frac{G_{\mu\nu}}{16\pi G}-\frac{1}{2}T_{\mu\nu}^{(m)}-\frac{1}{2}T_{\mu\nu}^{(\Phi)}\right]=0\ \ or\ \ \int_M d^4x=0. \tag{26}$$

But

$$\int_M d^4x\neq\ 0. \tag{27}$$

Hence,

$$\left[\frac{G_{\mu\nu}}{16\pi G}-\frac{1}{2}T_{\mu\nu}^{(m)}-\frac{1}{2}T_{\mu\nu}^{(\Phi)}\right]=0. \tag{28}$$

$$\therefore G_{\mu\nu}=8\pi G\left[T_{\mu\nu}^{(matter)}+T_{\mu\nu}^{(\Phi)}\right] \tag{29}$$

Now, (29) is the **modified Einstein field equation**.

3.3.7. Connect Residual Interactions to Curvature

By hypothesis,

$$\Phi=\Phi(F_R). \tag{30}$$

Therefore,

$$T_{\mu\nu}^{(\Phi)}=T_{\mu\nu}^{(\Phi(\mathrm{F_R}))}. \tag{31}$$

Hence,

$$F_R\longrightarrow\Phi\longrightarrow T_{\mu\nu}^{(\Phi)}\longrightarrow G_{\mu\nu}. \tag{32}$$

This chain shows that residual interactions influence spacetime curvature through the scalar field.

3.3.8. Demonstrate Curvature Generation

Supposing

$$F_R\neq 0. \tag{33}$$

Then

$$\Phi \neq 0. \tag{34}$$

Consequently,

$$\nabla_\mu \Phi \neq 0, \tag{35}$$

Or

$$V(\Phi) \neq 0. \tag{36}$$

Therefore,

$$T_{\mu\nu}^{(\Phi)} \neq 0. \tag{37}$$

Einstein's equations then imply;

$$G_{\mu\nu} = 8\pi G\left[T_{\mu\nu}^{(matter)} + T_{\mu\nu}^{(\Phi)}\right] \neq 0. \tag{38}$$

Hence,

$$G_{\mu\nu} \neq 0. \tag{39}$$

But, (39) means spacetime is curved.

Therefore, the residual interactions generate curvature.

Q.E.D.

4.0. Conclusion

Starting from the Einstein-Hilbert variational principle, it is demonstrated herein that:

1. Residual microscopic interactions produce a scalar field

$$F_R \rightarrow \Phi.$$

2. The scalar field possesses a stress-energy tensor

$$T_{\mu\nu}^{(\Phi)}.$$

3. This tensor enters Einstein's field equations

$$G_{\mu\nu} = 8\pi G\left[T_{\mu\nu}^{(matter)} + T_{\mu\nu}^{(\Phi)}\right].$$

4. Nonzero residual interactions imply nonzero curvature.

Therefore, Residual microscopic interactions induce effective spacetime curvature.

References

1. Einstein A, (1916). *The foundation of the general theory of relativity*. *Annalen der Physik, 49*(7), 769–822. https://doi.org/10.1002/andp.19163540702
2. Jacobson T. (1995). Thermodynamics of spacetime: The Einstein equation of state. *Physical Review Letters, 75*(7), 1260–1263. https://doi.org/10.1103/PhysRevLett.75.1260
3. Verlinde E. (2011). On the origin of gravity and the laws of Newton. *Journal of High Energy Physics, 2011*(4), 29. https://doi.org/10.1007/JHEP04(2011)029